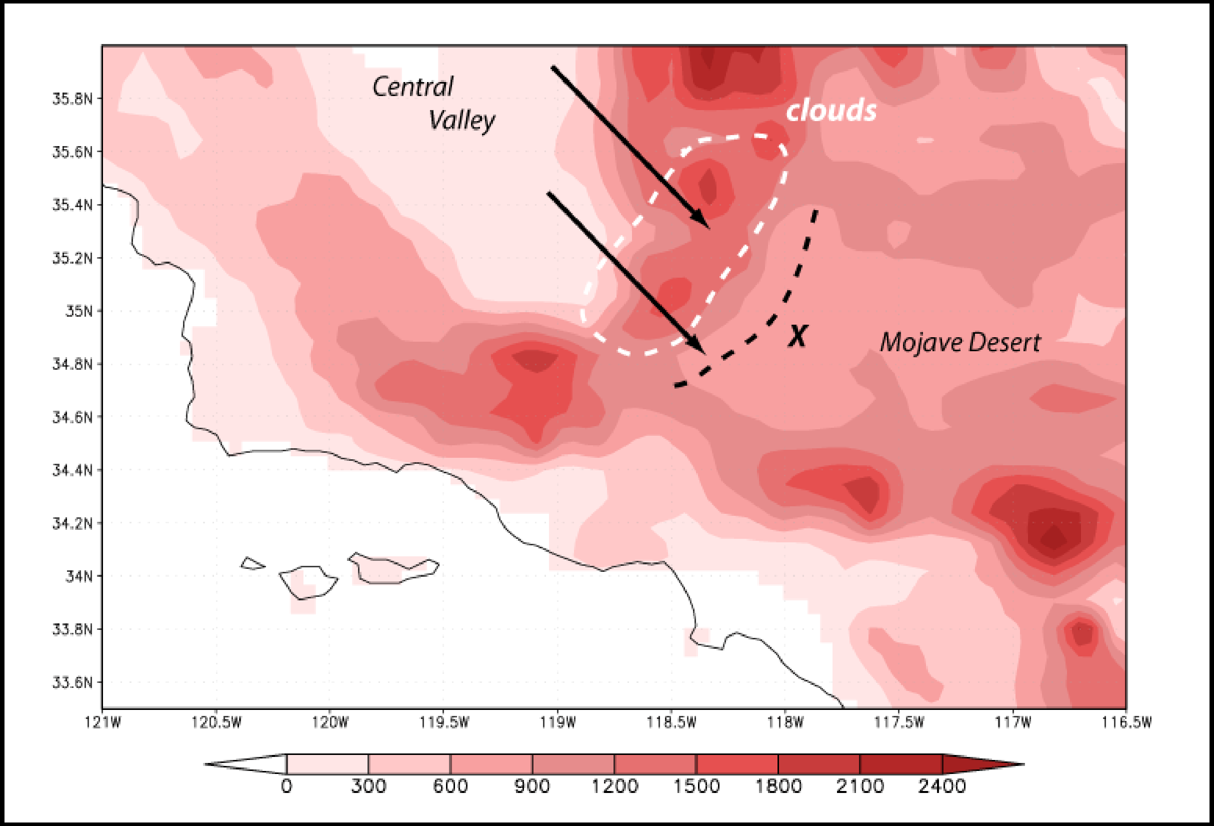
The hydraulic jump

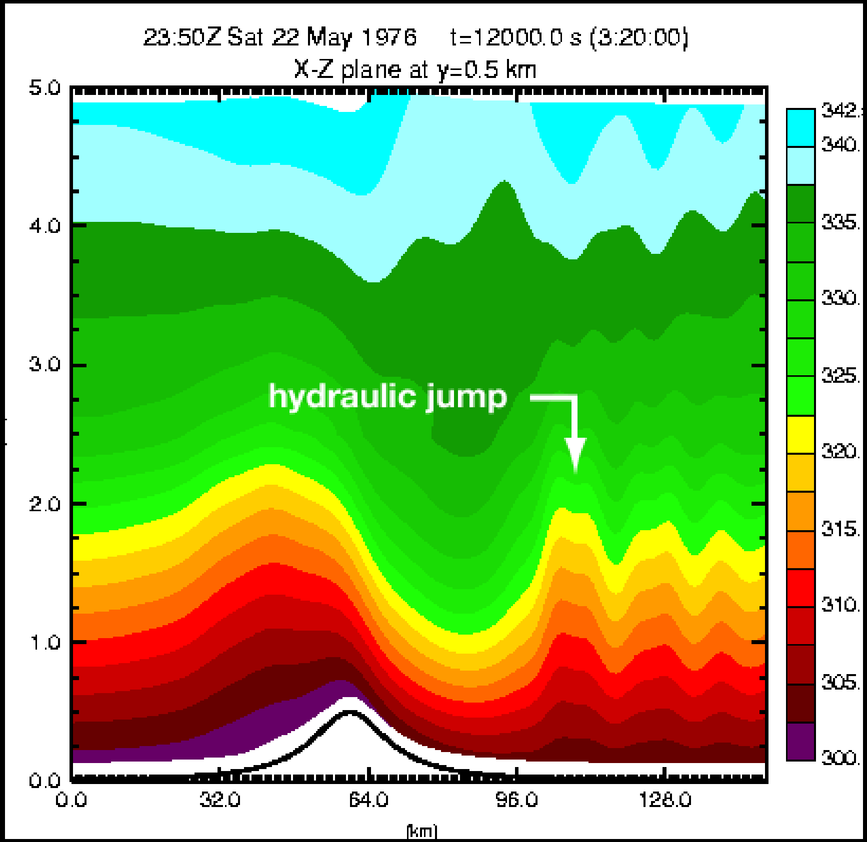
“As one watches them (clouds), they don’t seem to change, but if you look back a minute later, it is all very different.”

- Richard P. Feynman

Time-lapse cloud movie
(note calm foreground)

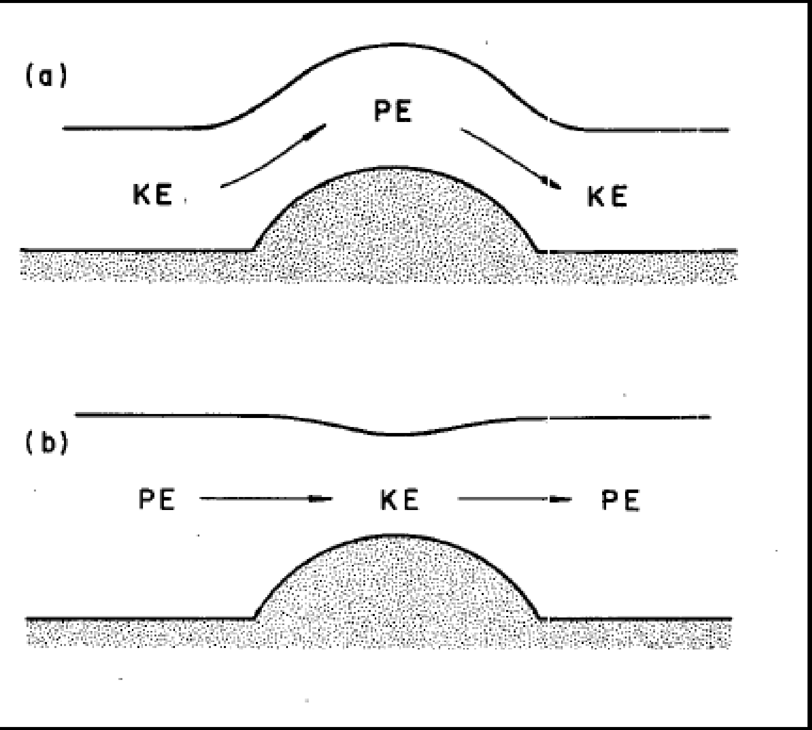
Topographic map





23:50Z Sat 22 May 1976 t=12000.0 s (3:20:00)
X-Z plane at y=0.5 km
hydraulic jump
5.0
4.0
3.0
2.0
1.0
0.0
0.0
32.0
64.0
96.0
128.0
(km)
342.
340.
335.
330.
325.
320.
315.
310.
305.
300.

Possible flows over obstacles

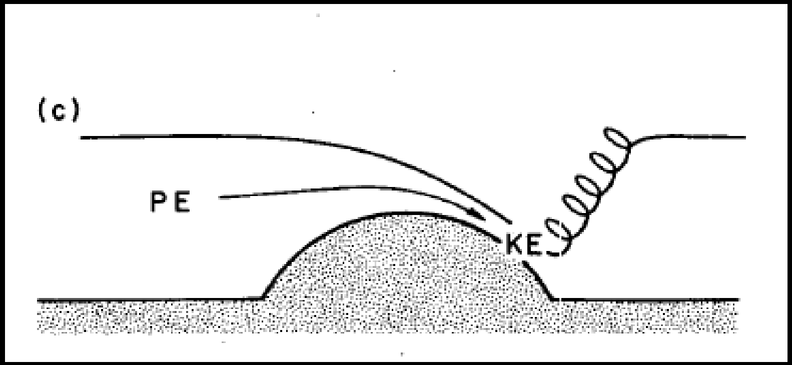


“supercritical flow”
(fluid thickens, slows over obstacle)

“subcritical flow”
(fluid thins, accelerates over obstacle)

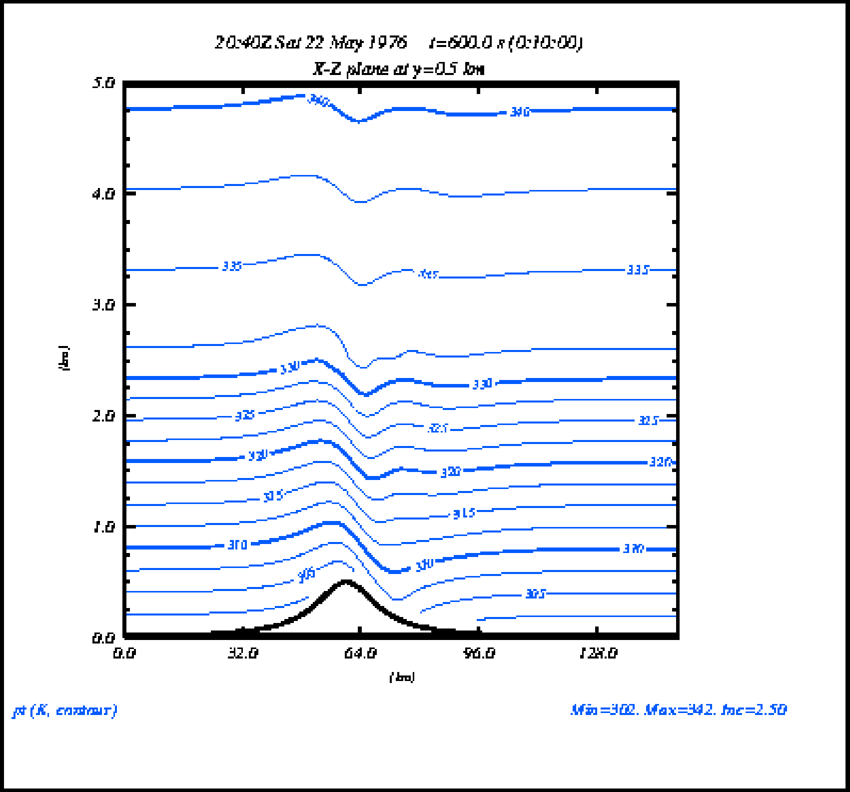
Durran (1986)

Hydraulic jump



Flow starts subcritical, accelerates over obstacle & suddenly becomes supercritical

Animation - potential temperature

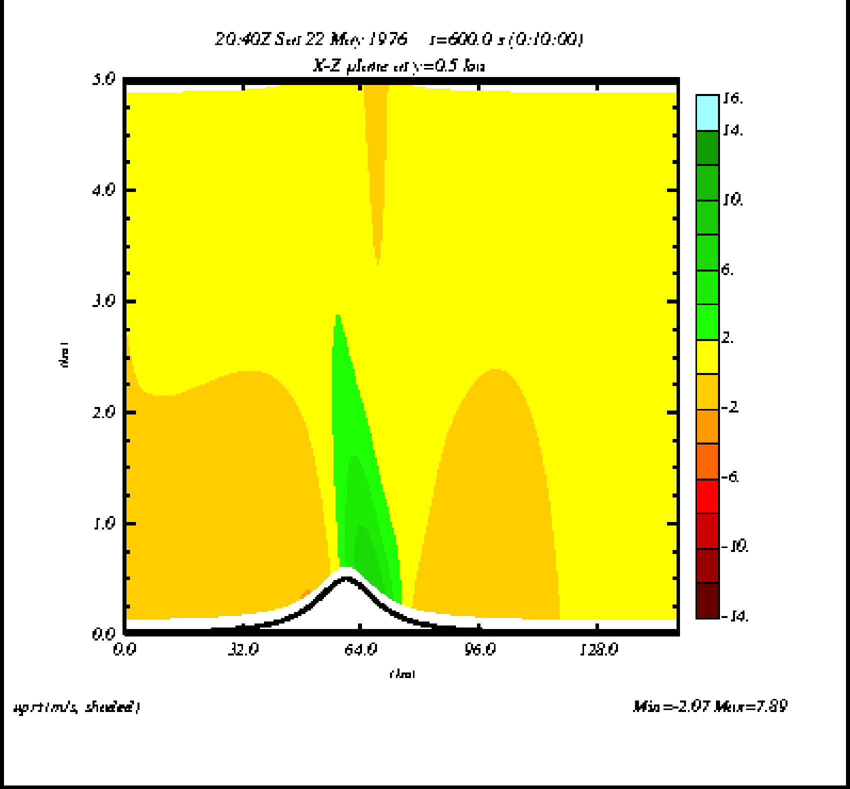


Adiabatic run so <u>isentropes</u> are <u>streamlines</u>

Note lower layer is more stable than upper layer

ARPS simulation

Animation - u’



20:40Z Sat 22 May 1976 t=600.0 s (0:10:00)
X-Z plane at y=0.5 km
5.0
4.0
3.0
2.0
1.0
0.0
0.0
32.0
64.0
96.0
128.0
16.
14.
10.
6.
2.
-2
-6.
-10.
-14.
upt (m/s, shaded)
Min=-2.07 Max=7.89

Hydraulic theory derivation

Highlights of derivation

$$\left[1 - \frac{u^2}{gh}\right] \frac{\partial h}{\partial x} = -\frac{\partial b}{\partial x}.$$

h=h(x) is fluid depth
b=b(x) is obstacle height

$$Fr^2 = \frac{u^2}{gh},$$

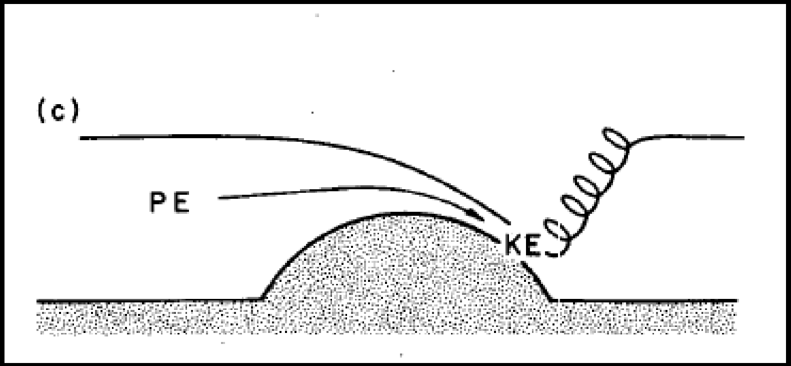
Froude number

Froude number dependence

Fr > 1 -- fluid thickens, slows on upslope
(supercritical flow)

Fr < 1 -- fluid thins, accelerates on upslope
(subcritical flow)

Fr < 1 transition to Fr > 1 over crest
--> hydraulic jump



Durran (1986)

Durran’s “Froude number”

$$Fr = \frac{\pi}{2}\frac{U}{H\sqrt{(N_L^2 - N_U^2)}}$$

U ↑ Fr ↑
H ↑ Fr ↓

For Fig. 3
U = 25 m/s (initial wind)
N_L = .025 (more stable lower layer)
N_U = .01 (less stable upper layer)
H = 3000 m (depth of lower stable layer)

Initial Froude number = 0.57 (subcritical)

Initial Fr = 0.57

Durran Fig. 3

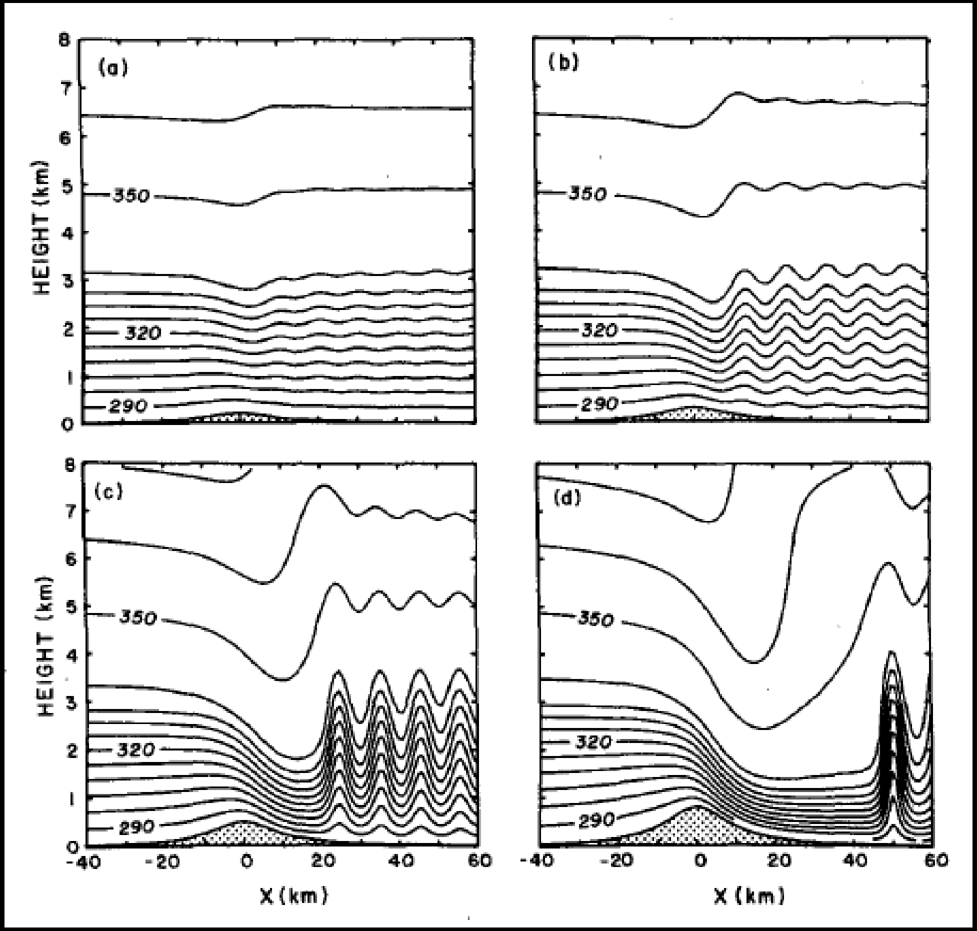
U = 25 m/s, H = 3000 m, vary mtn height

200 m mtn 300 m mtn

Fr at crest

Fr = 0.74
(Fr increased, but not by enough)

Fr = 1.19
(Fr increased by enough to become supercritical)



Fr at crest

Fr = 0.90
(Fr increased, but not by enough)

Fr = 1.27

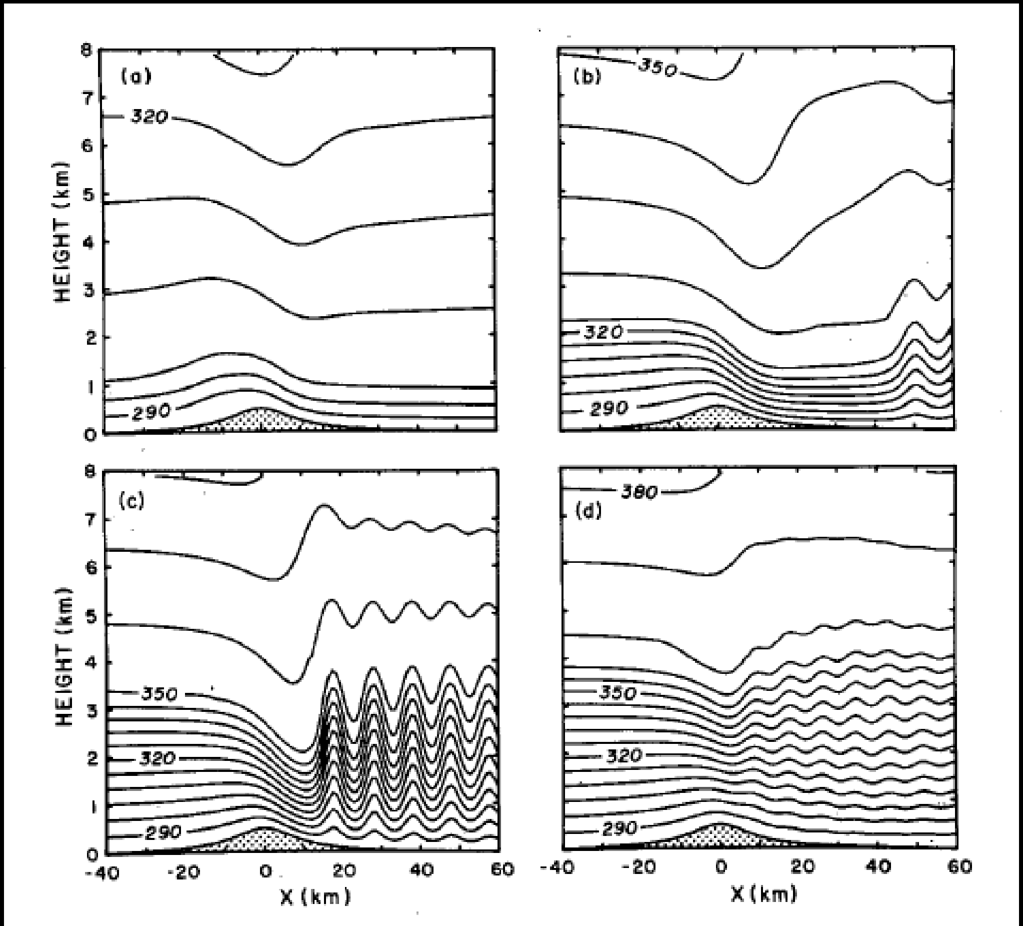
500 m mtn 800 m mtn

Durran Fig. 5

U = 25 m/s, 500 m mtn, vary H

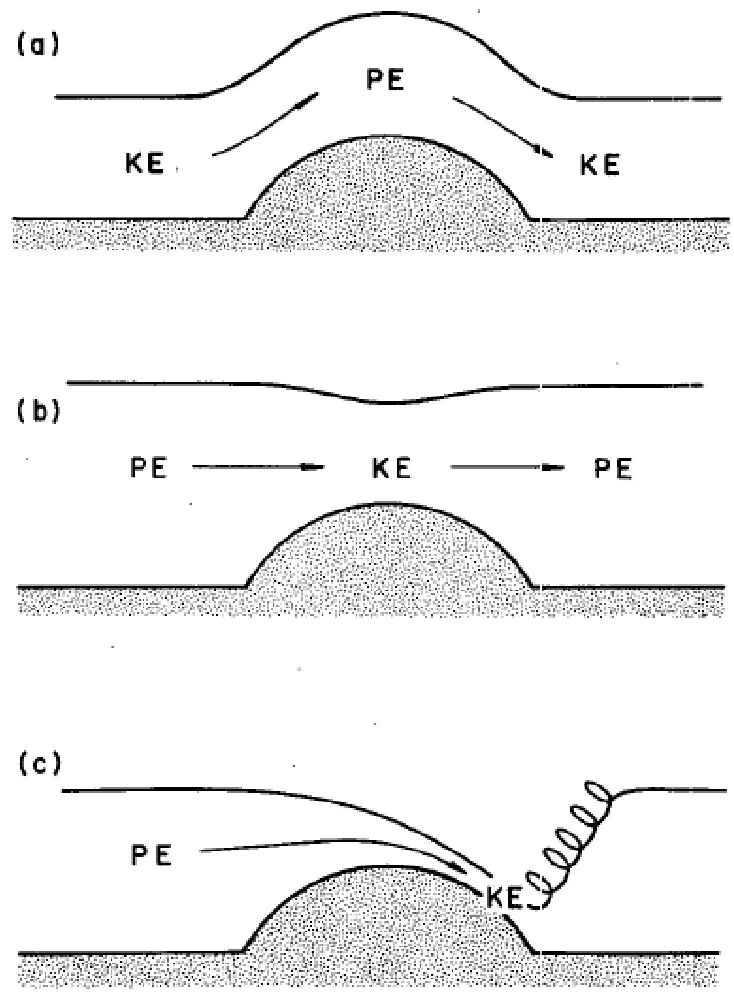
1000 m H 2500 m H

Fr > 1 everywhere
(fluid thickens upstream and thins downstream)



Fr < 1 Everywhere
(fluid thins upstream and thickens downstream)

3500 m H 4000 m H


(a)
PE
KE
KE
(b)
PE
KE
PE
(c)
PE
KE